ISI – Bangalore Center – B Math - Physics II – Back paper Exam
Date: 1January 2019. Duration of Exam: 3 hours
Total marks: 60

ANSWER ALL QUESTIONS

Q1. [Total Marks: 2+4+4=10]

a.) Explain the process of "adiabatic free expansion" of a gas by means of an example.

b.) In the adiabatic free expansion of an ideal gas, it was experimentally found that there was no change of temperature. Show that this implies that for an ideal gas

$U(V,T) = \int C_v(T)dT +$ a constant where $U(V,T)$ is the internal energy where C_v is the heat capacity at constant volume.

c.) Calculate the change of entropy of one mole of an ideal gas in an adiabatic free expansion where the volume changes from V to $2V$.

Q2. [Total Marks: 2+4+4=10]

A body is initially kept at thermal equilibrium with a heat reservoir at temperature $T_{1.}$ It is then placed in a refrigerator which, working in cycles, reduces the temperature to T_2 by dumping heat into the reservoir. Assume that the entropy of the body being cooled changes from $S_{1.}$ to $S_{2.}$ Also assume that the volume of the body does not change.

a.) What is the change of entropy of the refrigerator?
b.) What is the change of entropy of the reservoir?
c.) Show that the minimum amount of energy that has to be supplied to the refrigerator is $T_1(S_1 - S_2) - C(T_1 - T_2)$ where C is the heat capacity of the body at constant volume.

Q3. [Total Marks: 4+4+4+3=15]

a.) Using only the first law of thermodynamics show that for any gas that can be described by a PVT system

$C_p - C_v = \left[\left(\frac{\partial U}{\partial V}\right)_p + P\right]\left(\frac{\partial V}{\partial T}\right)_p$ where U is the internal energy $C_p{}'C_v$ are heat capacities.

b.) Using the second law of thermodynamics, and Maxwell relations show that the above can be written as

$$C_p - C_v = T\left(\frac{\partial P}{\partial T}\right)_P \left(\frac{\partial V}{\partial T}\right)_p$$

c.) Show that $\left(\frac{\partial P}{\partial T}\right)_p = -\left(\frac{\partial P}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_p$.

d.) Prove that $C_p - C_v \geq 0$.

Q4. [Total Marks: 4+5+6=15]

a.)Using the partition function Z of a generic statistical mechanical system

i) Show that $< E > = -\frac{\partial \ln Z}{\partial \beta}$.

ii) Find a relationship between $C_v = \frac{\partial}{\partial T} < E >$ and $< (E - < E >)^2 >$ and show that it implies that C_v cannot be negative.

b.) Write the expression for the partition function for a collection of mono atomic ideal gas with no internal degrees of freedom. Show that $< E > = \frac{3}{2} NkT$ where N is the number of atoms present.

Q5. [Total Marks: 5+5=10]

a.)A soap bubble 250 nm thick is illuminated with white light. The index of refraction of the soap film is 1.36. Which wavelengths are *not* seen in the reflected light? Which wavelengths appear strong in the reflected light? What colour does the soap film appear at normal incidence? For your reference the visible light spectrum is given.

b.)In Young's double slit experiment, a thin mica sheet (n = 1.5) is introduced in the path of one of the beams. If the central fringe gets shifted by 1.2 cm, calculate the thickness of the mica sheet, Assume that the separation between the slits is 0.1 cm and the distance between the slits and the screen is 50 cm.

Maxwell Relations that you may use or not use:

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V$$

$$\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$$

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$$

$$\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P$$